Adaptive Pulse Compression Repair Processing

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ABSTRACT

It is well known that the standard matched filter used in radar pulse compression generates range sidelobes in the vicinity of large targets which can result in the masking of smaller nearby targets. Recently, Pulse Compression Repair (PCR) was proposed as a means of suppressing the range sidelobes after matched filtering has taken place. This is especially applicable to in-service radar systems, where one may not have access to the received signal prior to standard pulse compression or where it may not be feasible to replace the current pulse-compression system. The PCR algorithm adaptively operates on the output of the standard matched filter, thereby treating the autocorrelation of the received waveform as if it were the transmitted waveform. This paper examines the effects of the Doppler mismatch on the PCR algorithm in which the resulting correlation between the transmitted waveform and the Doppler-shifted received waveform is mismatched to the waveform autocorrelation. It is shown that PCR degrades gracefully as Doppler mismatch increases, even in dense target scenarios.

1. INTRODUCTION

The standard pulse compression matched filter [1] is known to maximize the received signal-to-noise ratio (SNR) for a solitary point target in the presence of white noise. However, the matched filter suffers from range sidelobes that are dependent upon the autocorrelation (or in general the ambiguity function) of the transmitted waveform. Hence, in the vicinity of large targets the range sidelobes may mask the presence of other small targets. Furthermore, range sidelobes hinder Constant False Alarm Rate (CFAR) detectors by falsely raising the quiescent noise floor.

To suppress range sidelobes, other deterministic alternatives to the matched filter have been devised, which include optimum mismatched filters [2,3] and Least Squares estimation [4,5]. Gabriel [6,7] was the first to consider applying pulse compression in an adaptive manner by computing a sample covariance matrix from the returns of many pulses in order to achieve higher range resolution for Inverse Synthetic Aperture Radar (ISAR). More recently, based on the received signal from a single pulse, the Adaptive Pulse Compression (APC) [8-11] approach adaptively determines an appropriate receiver pulse-compression filter to use for each individual range cell – thereby applying a receive filter that is truly "matched" to the received signal. The APC algorithm is based upon a Minimum Mean-Square Error

(MMSE) formulation [12] whereby, for a given range cell, nulls are placed at the relative range offsets associated with large targets identified by the matched filter.

In many in-service radar systems the received signal prior to pulse compression is not available or it is simply not feasible to replace the current pulse compression system. In this case, the original APC approach is not appropriate. However, a modified version of APC denoted as Pulse Compression Repair (PCR) [13,14] is quite well-suited to in-service radar systems. The PCR approach applies the APC methodology *after* standard matched filtering whereby the transmitted waveform in the formulation is replaced by its autocorrelation (as a result of matched filtering).

This paper addresses Doppler mismatch as it pertains to the PCR algorithm. In this case the expected autocorrelation (*i.e.* the "effective waveform") will be mismatched to the actual waveform - matched filter correlation. The result is some loss in SNR as well as an increase in the range sidelobes surrounding the Doppler-mismatched target. The increased range sidelobes occur because the Doppler mismatch hinders the cancellation of the interference from the large targets at the surrounding range cells. It is shown that despite the degradation caused by Doppler mismatch the PCR algorithm is greatly superior to the matched filter, especially in dense target environments.

2. RECEIVED SIGNAL MODEL

To establish the received signal model for the PCR algorithm we begin by formulating the matched filter signal model. In the digital domain the range profile illuminated by the radar can be represented as a discrete impulse response with which the transmitted waveform convolves. Thus, matched filtering of the received return signal can be expressed as

$$\hat{x}_{MF}(\ell) = \mathbf{s}^H \mathbf{y}(\ell), \qquad (1)$$

where $\hat{x}_{MF}(\ell)$, for $\ell = 0, \cdots, L-1$, is the estimate of the ℓth range cell within the range window of interest, $\mathbf{s} = [s_1 \ s_2 \ \cdots \ s_N]^T$ is the length-N sampled version of the transmitted waveform, $\mathbf{y}(\ell) = [y(\ell) \ y(\ell+1) \ \cdots \ y(\ell+N-1)]^T$ is a vector of N contiguous received radar return samples, and $(\bullet)^T$ and $(\bullet)^H$ are the transpose and conjugate transpose (or

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14. ABSTRACT

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Hermitian) operations, respectively. Each individual radar return sample can be expressed as

$$y(\ell) = \mathbf{x}^T(\ell)\,\mathbf{s} + v(\ell), \tag{2}$$

where $\mathbf{x}(\ell) = [x(\ell)\ \ x(\ell-1)\ \cdots\ x(\ell-N+1)]^T$ consists of N samples of the true range profile impulse response and $v(\ell)$ is additive noise. The matched filter output of (1) can therefore be rewritten as

$$\hat{x}_{MF}(\ell) = \mathbf{s}^H \mathbf{A}^T(\ell)\,\mathbf{s} + \mathbf{s}^H \mathbf{v}(\ell), \tag{3}$$

where $\mathbf{v}(\ell) = [v(\ell)\ \ v(\ell+1)\ \cdots\ v(\ell+N-1)]^T$ and

$$\mathbf{A}(\ell) = \begin{bmatrix} x(\ell) & x(\ell+1) & \cdots & x(\ell+N-1) \\ x(\ell-1) & x(\ell) & \ddots & \vdots \\ \vdots & \ddots & \ddots & x(\ell+1) \\ x(\ell-N+1) & \cdots & x(\ell-1) & x(\ell) \end{bmatrix} \tag{4}$$

is a collection of sample-shifted snapshots (in the columns) of the range profile impulse response. From (4), it is apparent that whenever any of the off-diagonal elements of $\mathbf{A}(\ell)$ are relatively large, the estimation of $x(\ell)$ may be masked by range sidelobes.

The received signal model in (3) and (4) was employed to develop the APC algorithm such that an MMSE filter could be estimated for each individual range cell to suppress range sidelobes. Range sidelobe suppression can also be achieved via adaptive post-processing of the matched filter output by expressing the received signal model in terms of the waveform autocorrelation. This is possible because the operations of convolution of the transmitted waveform with the radar impulse response in (2) and the convolution of the received return signal with the time-reversed, complex-conjugated waveform in (1) can be combined such that (3) is re-expressed as

$$\hat{x}_{MF}(\ell) = \tilde{\mathbf{x}}^T(\ell)\,\tilde{\mathbf{r}} + u(\ell) \tag{5}$$

where $\tilde{\mathbf{x}}(\ell) = [x(\ell+N-1)\ \cdots\ x(\ell+1)\ x(\ell)\ x(\ell-1)\ \cdots\ x(\ell-N+1)]^T$ consists of $2N-1$ samples of the true range profile impulse response, $u(\ell)$ is additive noise correlated with the matched filter, and $\tilde{\mathbf{r}}$ is the length $2N-1$ autocorrelation of the transmitted waveform $\mathbf{s}$. We treat the matched filter output as the received return signal (as in (2)) by collecting $2N-1$ contiguous samples of the matched filter output $\hat{x}_{MF}(\ell)$ from (5) into $\tilde{\mathbf{y}}(\ell) = [\hat{x}_{MF}(\ell-N+1)\ \cdots\ \hat{x}_{MF}(\ell)\ \cdots\ \hat{x}_{MF}(\ell+N-1)]^T$ which can be expressed as

$$\tilde{\mathbf{y}}(\ell) = \mathbf{B}^T(\ell)\,\tilde{\mathbf{r}} + \tilde{\mathbf{u}}(\ell), \tag{6}$$

where $\tilde{\mathbf{u}}(\ell) = [u(\ell-N+1)\ \cdots\ u(\ell)\ \cdots\ u(\ell+N-1)]^T$ and

$$\mathbf{B}(\ell) = \begin{bmatrix} x(\ell) & x(\ell+1) & \cdots & x(\ell+2N-2) \\ x(\ell-1) & x(\ell) & \ddots & \vdots \\ \vdots & \ddots & \ddots & x(\ell+1) \\ x(\ell-2N+2) & \cdots & x(\ell-1) & x(\ell) \end{bmatrix} \tag{7}$$

is a $(2N-1)\times(2N-1)$ matrix of sample-shifted snapshots of the radar impulse response. Then, based upon a formulation similar to APC we determine the appropriate MMSE receive filter $\tilde{\mathbf{w}}(\ell)$ for each individual range cell that is to be applied as

$$\hat{x}_{PCR}(\ell) = \tilde{\mathbf{w}}^H(\ell)\,\tilde{\mathbf{y}}(\ell). \tag{8}$$

The following section briefly describes the development of the MMSE receive filter $\tilde{\mathbf{w}}(\ell)$ for the PCR algorithm.

3. *PULSE COMPRESSION REPAIR*

The matched filter output $\tilde{\mathbf{y}}(\ell)$ is adaptively pulse compressed with the respective MMSE receive filter obtained by minimizing the MMSE cost function [12]

$$J(\ell) = E\left[\left|x(\ell) - \tilde{\mathbf{w}}^H(\ell)\,\tilde{\mathbf{y}}(\ell)\right|^2\right] \tag{9}$$

where $\tilde{\mathbf{w}}(\ell)$ is the length $2N-1$ MMSE receive filter specific to the estimation of range cell $x(\ell)$. Solving (9) for $\tilde{\mathbf{w}}(\ell)$ yields the PCR filter for each individual range cell as

$$\tilde{\mathbf{w}}(\ell) = \hat{\rho}(\ell)\,(\mathbf{C}(\ell) + \mathbf{R})^{-1}\,\tilde{\mathbf{r}} \tag{10}$$

where

$$\hat{\rho}(\ell) = |\hat{x}(\ell)|^2 \tag{11}$$

is the estimated power of $x(\ell)$, $\mathbf{R} = E\left[\mathbf{u}(\ell)\,\mathbf{u}^H(\ell)\right]$ is the covariance matrix of the noise correlated with the matched filter, and the matrix $\mathbf{C}(\ell)$ is defined as

$$\mathbf{C}(\ell) = \sum_{n=-2N+2}^{2N-2} \hat{\rho}(\ell-n)\,\tilde{\mathbf{r}}_n\,\tilde{\mathbf{r}}_n^H \tag{12}$$

where $\tilde{\mathbf{r}}_n$ contains the elements of the length $2N-1$ waveform autocorrelation $\tilde{\mathbf{r}}$ shifted by n samples and the remainder zero filled. For example, $\tilde{\mathbf{r}}_2 = [0\ 0\ r_{-N+1}\ \cdots\ r_{N-3}]^T$ for $n=2$ and $\tilde{\mathbf{r}}_{-2} = [r_{-N+3}\ \cdots\ r_{N-1}\ 0\ 0]^T$ for $n=-2$. Also, assuming that the noise power is small compared to the power of the radar returns and that the waveform has relatively good

autocorrelation properties (sufficiently low sidelobe levels), the noise covariance matrix $\mathbf{R}$ can be approximated as $\sigma_v^2 \mathbf{I}$, where σ_v^2 is the noise power. The noise power can be assumed known since internal thermal noise is known to dominate the external noise at microwave frequencies (where most radars operate) [1].

As (10)-(12) illustrate, the PCR receive filter for a particular range cell is a function of the current power estimate of the range cell of interest as well as the surrounding range cells. Initial estimates of the range cell powers can be obtained from the normalized output of the matched filter as

$$\hat{\rho}_0(\ell) = \left| \frac{\hat{x}_{MF}(\ell)}{\|\mathbf{s}\|^2} \right|^2 . \tag{13}$$

The power estimates from (13) are inserted into (10) and (12) to generate the respective PCR receive filters which are then applied onto the corresponding $\tilde{\mathbf{y}}(\ell)$ to re-estimate the complex range cell amplitudes. This process is repeated, with the range cell power estimates obtained from the previous stage, for a predetermined number of stages. Note that the PCR algorithm need only be applied when a large target is present which would therefore generate range sidelobes.

4. NUMERICAL STABILITY

The matrix $\mathbf{C}(\ell) + \mathbf{R}$ could potentially become ill-conditioned in the vicinity of very large targets or when small range cell estimates approach zero. However, the same heuristic approach described in [13] for APC will also work for PCR which is to replace $\hat{\rho}(\ell) = |\hat{x}(\ell)|^2$ with $\hat{\rho}(\ell) = |\hat{x}(\ell)|^\alpha$ and (under the white noise assumption) replacing the noise power σ_v^2 in (10) with σ_v^α, for $0 \le \alpha \le 2$. For the case of large SNR targets, using $\alpha < 2$ reduces the effective SNR dynamic range and thereby alleviates the possibility of ill-conditioning. It has been found based upon extensive simulation that use of values of $1.4 \le \alpha \le 1.7$ with 1 or 2 stages of the PCR algorithm tend to yield the best results. Furthermore, α should be set at the high end (near 1.7) initially to quickly drive down the sidelobes from large SNR targets and then decrease. It is a topic of future research to determine if optimal values of α can be found as a function of the surrounding range cell estimates.

An additional heuristic approach that can be used to alleviate ill-conditioning is to set a lower bound upon the magnitudes of the range cell estimates. This is done so that slightly larger values of α can be used to drive down the sidelobes from large targets more quickly without driving smaller range cell estimates to zero.

5. SIMULATION RESULTS

We consider two scenarios: 1) A single large target return that masks a nearby smaller return, and 2) a dense target scenario containing several targets with disparate power levels. For each scenario we examine the case of no Doppler for the target(s) and compare this with the case in which the target(s) have high Doppler mismatch. In all cases, the length $N = 30$ Lewis-Kretschmer P3 code [15] is the transmitted waveform (although any waveform with sufficient sidelobe levels can be used). The noise power is set at -70 dB with respect to the largest target. Note that in the following figures the matched filter outputs are normalized by N in order to make a fair comparison.

For the first scenario we consider a large target in the proximity of a smaller target. A single stage of the PCR algorithm is employed using $\alpha = 1.6$. As shown in Fig. 1, the sidelobes from the large target generated by standard matched filtering completely mask the presence of the smaller target. However, by applying PCR to the matched filter output the sidelobes are suppressed by roughly 40 dB so that the PCR output closely matches the range profile ground truth and the small target is now easily visible. In terms of Mean-Square Error (MSE), over the range window of interest the matched filter achieves an MSE of -29 dB and a single stage of PCR reduces that MSE to -64 dB, a 35 dB improvement. In other words, PCR greatly enhances the range profile estimation accuracy.

To examine the effects of Doppler mismatch on the first scenario we apply a 3° phase shift over the length of the waveform for the large target. This Doppler phase shift is analogous to Mach-1 target illuminated by a 1µs S-band pulse. As Fig. 2 illustrates, the matched filter output experiences no noticeable change and is found to achieve the same MSE level as when no Doppler is present. However, due to its significantly increased sensitivity the PCR algorithm (a single stage) has some degradation that manifests in the form of increased range sidelobe levels. The MSE for PCR now has increased to -45 dB which is 19 dB higher than when no Doppler is present and is predominantly due to the sidelobe "shoulders" within ±10 range cells around the large target. Note, though, that PCR is still considerably superior to the matched filter and that the smaller target is still visible using PCR.
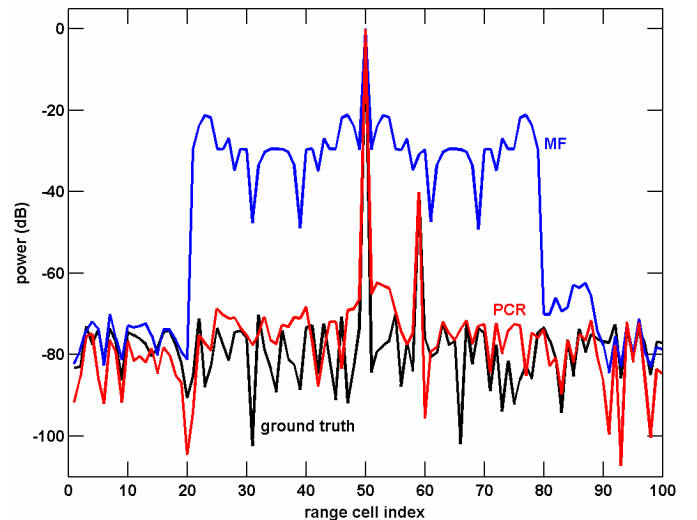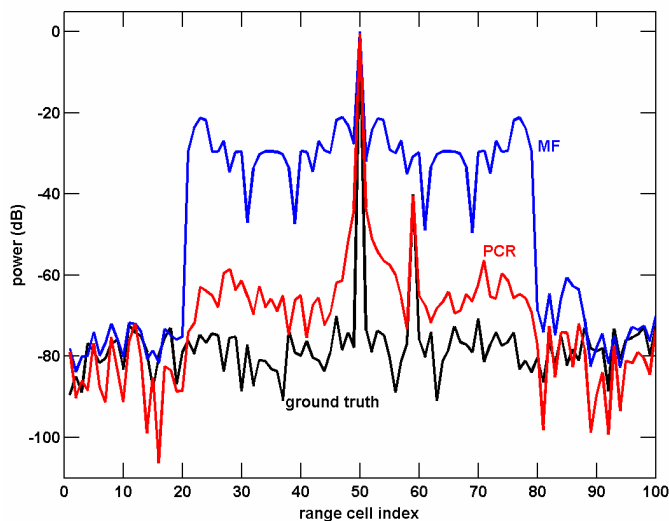

Fig. 1. Sparse targets with no Doppler

Fig. 2. Sparse targets with high Doppler

Fig. 3. Dense targets with no Doppler

Fig. 4. Dense targets with high Doppler

For the dense target scenario we consider 100 range cells within which 12 targets of various powers are present. The smallest target, visible in Fig. 3 at range cell index 65, is masked by the largest target, located at range cell index 83, which has a 52 dB higher return power. In fact, using the standard matched filter, it is found that the 8 targets at range cell indices 9, 36, 40, 55, 65, 88, 92, and 97 would all be masked to some degree and undetectable by means of a CFAR detector. On the contrary, two stages of the PCR algorithm (using $\alpha = 1.6$ and 1.4, respectively) suppress the range sidelobes nearly all the way to the noise floor so that all of the small targets are visible. In terms of MSE, while the matched filter achieves -29 dB, the PCR algorithm yields a 42 dB improvement with -71 dB.

To ascertain the effects of Doppler mismatch on the dense target scenario a random Doppler shift over the length of the waveform taken uniformly from ±3° is applied to every range cell. Furthermore, the two largest targets at range cell indices of 29 and 83 have Doppler shifts set at +3° and -3°, respectively, as Doppler mismatch on the largest targets has been found to be the most detrimental. As with the sparse case, the matched filter output has no noticeable effect yet the increased sensitivity of PCR results in a general increase in range sidelobe levels. Also, the sidelobe shoulders are again visible around the two largest targets and a marked increase in sidelobes levels can be seen around range cell 60 which happens to be where the range sidelobes from the two largest targets overlaps. In terms of MSE, the matched filter remains the same with PCR now achieving -50 dB; a 21 dB loss over the no-Doppler case yet still 21 dB better than the matched filter.
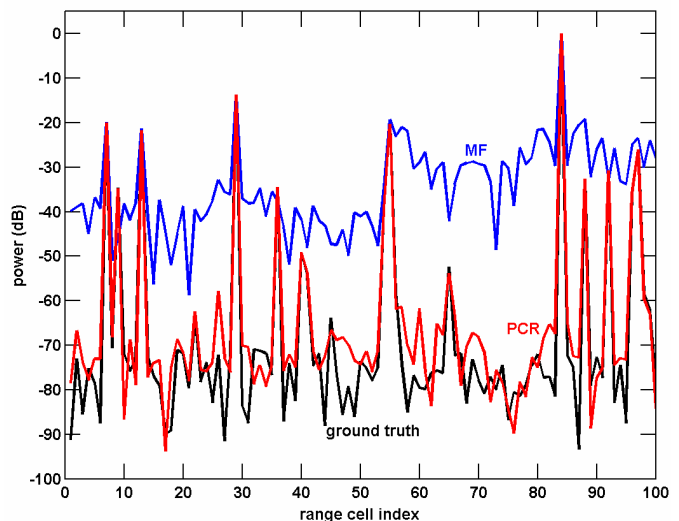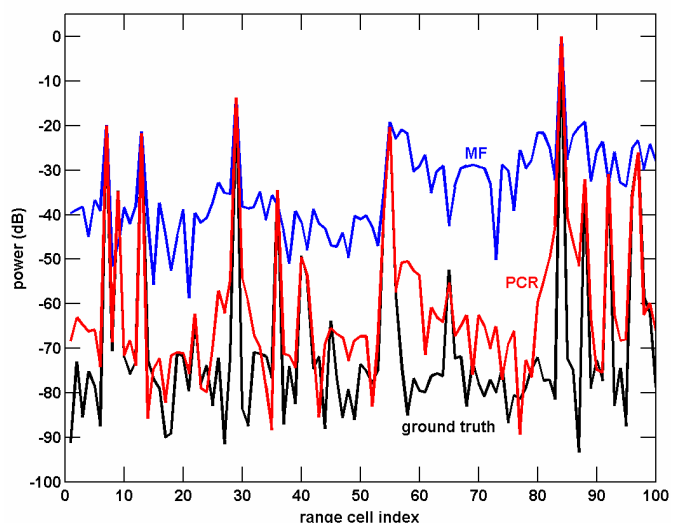
CONCLUSIONS

For many in-service radar systems, it is not feasible to replace the current pulse-compression apparatus. Therefore, the recently developed APC (Adaptive Pulse Compression) algorithm is not applicable. However, by treating the output of the matched filter as the received signal, a variant of APC, denoted Pulse Compression Repair (PCR), can be applied in which the "waveform" to which matching is applied is now the autocorrelation of the transmitted waveform. The PCR algorithm has been found to suppress the range sidelobes of the matched filter nearly to the noise floor and to reduce the Mean-Square Error of the range profile estimate by orders of magnitude. Furthermore, PCR has been shown to degrade gracefully in the presence of target Doppler as a result of increased range sidelobes, the largest of which is manifested as sidelobe shoulders very close to the Doppler-mismatched target. Therefore, even for relatively large Doppler mismatches, the degradation tends to be localized, thus minimizing any adverse effects.

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